

# A mechanical model of a PR-Box

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## Abstract

Several mechanical realizations of PR-Boxes are discussed. Apart from superluminal correlations, which are known not to be realizable by classical devices, the requirements (1) PR-condition on input-output relations, (2) non-signaling, and (3) allowing for independent measurements are discussed separately. The examples show that internal signaling (inside the box) does not invariably imply the possibility of external signaling and may externally be attributed to contextuality. The proposed model requires two classical bits to be exchanged internally in order to realize the defined requirements. However, with EPR correlations this can be reduced to one classical bit.

## 1 Introduction

In a seminal article, Popescu and Rohrlich [9] have shown in 1992 that a maximal violation of the CHSH inequality (for CHSH inequalities see, e.g., [4, 5]) is possible with a simple device which became known as a PR-box (for a recent review see [10]). Essentially, a PR-box is a black-box device on which two persons – Alice and Bob – can make local measurements with binary variables as input (let  $x$  be the binary input of Alice and  $y$  the binary input of Bob) and binary outputs  $a$  (at Alice’s side) and  $b$  (at Bob’s side), such that

$$a + b = x \cdot y \pmod{2}. \quad (1)$$

This implies that unless  $x$  and  $y$  are both equal to 1, the outputs  $a$  and  $b$  are equal (either both 0 or both 1), and if  $x = y = 1$  the outputs  $a$  and  $b$  are different.

The fact that quantum physics is more nonlocal than classical physics but not as nonlocal as a PR-Box (this has sometimes been called superstrong nonlocality) led to a completely new approach of axiomatizing quantum theory (see,

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e.g., [2, 8]. The main question in this approach is: Can we define quantum theory by the degree to which non-local correlations can be used to reduce certain communication problems. Some of the particular interesting consequences of superstrong non-locality, like e.g. the reduction of the “inner product problem” have been analyzed in [12].

In addition to the property expressed by eq. (1) one generally requires that the output of the PR-box cannot be used for signaling between Alice and Bob. This implies that the probability for output  $b$  being 1 or 0 should not depend on the input  $x$  of Alice and, vice versa, the probability distribution for  $a$  should not depend on the input of Bob. Let  $P(a|x, y)$  be the conditional probability for  $a$  given  $x$  and  $y$ , and similarly  $P(b|x, y)$  the conditional probability for  $b$  given  $x$  and  $y$ , then this implies

$$P(a|x, y) = P(a|x) \quad \text{and} \quad P(b|x, y) = P(b|y). \quad (2)$$

This condition has also been called “marginal selectivity” [6]. It implies non-signaling and is sometimes considered as a necessary condition for a system to be consistent with special relativity. If this condition is violated and signaling possible, this signaling should be bound to local causality, i.e., it should not propagate faster than the speed of light.

As a third condition one often requires that Alice and Bob can perform their measurements independently in the sense that both receive their output immediately after they entered their input, i.e., neither of them has to wait for the other to perform his or her experiment.

One natural question with respect to such a device is whether it can exist or not. It can be shown that with such a device one can violate the so-called CHSH inequality maximally (see, e.g. [12, 1, 7]). Therefore, a device based on classical laws of physics cannot exist, and even quantum physics with its non-localities cannot realize a maximal violation of the CHSH expression. However, if we allow for an internal exchange of information (and thus give up the possibility that Alice and Bob perform their experiments within each others causal complements of the light cones corresponding to the events of measurements), one can show that such a device can be realized by a classical electronic circuit. It is important to notice, however, that such a device, despite the fact that internally an exchange of information is happening, cannot necessarily be used for signaling externally.

The question whether such a device can be built or not may be of relevance as it has been suggested that a PR-Box may be a logical contradiction, i.e. cannot exist for principal logical reasons, and under quite general conditions this can even be proven ([11]). In addition, in this letter I will raise the question, how many bits have to be exchanged internally within the PR-box in order to realize such a system. The device introduced in this letter shows that internally the exchange of two classical bits is sufficient. However, without giving a proof (which I don’t have), it suggests that this may also be necessary in order to fulfill the no-signaling condition. Furthermore, if we replace the random number generator used in this device by EPR measurements, this may be reduced to one classical bit.

In the next section 2, I will introduce three types of PR-boxes of increasing complexity and in section 3, I will discuss the internal signaling of these PR-boxes. In particular, in this section I will discuss the number of bits which need to be exchanged internally. A brief summary ends the article.

## 2 Signaling an non-signaling PR-boxes

In this section, I will construct three different PR-boxes such that the final one will come as close as possible to the abstract notion of a PR-box.

1. The first PR-box will satisfy eq. (1), but it can be used for signaling from Bob to Alice.
2. The second PR-box will also satisfy eq. (1), however, external signaling is no longer possible. This PR-box is asymmetric in the sense that Bob can perform his measurements independent of Alice, but Alice has to wait for her result until also Bob has performed his measurement.
3. Finally, I construct a symmetric box which satisfies eq. (1), which cannot be used for external signaling, and both partners can perform their measurements independently and obtain an immediate output on their respective sides.

As mentioned, the second and the third PR-box cannot be used for signaling. Of course, this does not imply that there are no signals exchanged internally. Therefore, I distinguish between external signaling, which refers to the possibility of Alice and Bob to use this device for an exchange of information, and internal signaling which is happening inside the box but may not be utilized by Alice and Bob.

### 2.1 The signaling PR-box

Figure 1 shows a simple circuit which realizes equation 1. The output on Bob's side is always 0, independent of his input. His input just triggers a device which generates 0 for his output.

The output on Alice's side depends on both inputs: only when Alice and Bob "throw in" a 1, the output will be 1 on her side, in all other cases it will be 0. The output on her side is realized by a simple logical AND-gate which leads to eq. (1). While Bob can obtain his output immediately after he inserted his input, Alice has to wait for Bob to enter his input, otherwise the AND-gate cannot produce an answer.

Even though eq. 1 is realized, this box can be used for signaling, at least from Bob to Alice. If Alice always uses  $x = 1$  as input, then obviously her output will be exactly the same as Bob's input: 0 if Bob uses 0 as input and 1 if Bob uses 1 as input. There is a strict correlation between the output on Alice's side with the input on Bob's side.

Let us finally remark that in this device internally one bit of information is signaled from Bob to Alice.

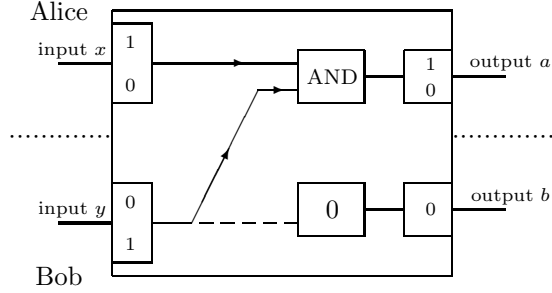


Figure 1: A simple circuit which gives the output of a PR-box. The dashed line does not carry bits but just triggers a device which generates a 0. This circuit can be used for signaling from Bob to Alice.

## 2.2 A non-signaling PR-box

Now let us consider Fig. 2. This circuit differs from the previous one in that the device which simply generated a 0 on Bob's side has now been replaced by a random number generator (RNG). This random number generator not only determines the output on Bob's side (which now is random) but it also adds this random number to the output of the AND-gate on Alice's side: essentially, it switches the output if the random number is 1 and it leaves the output unchanged if the random number is 0. This is happening in the “+RN” (“add random number”)-device.

Obviously, the outputs on both sides are always equal, unless both use 1 as an input, in which case the outputs are different. Therefore, this device still satisfies eq. (1). However, due to the random number generator, this box cannot be used for external signaling, as statistically the output on Alice's side is no longer correlated with the input on Bob's side.

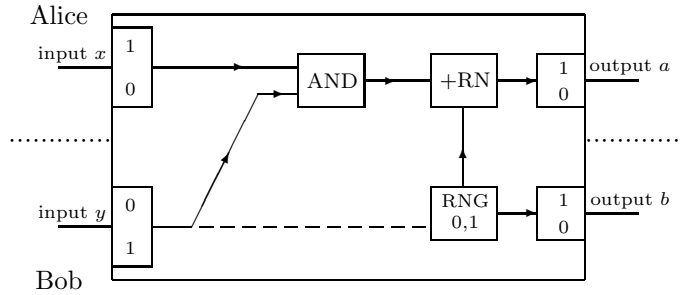


Figure 2: The output of this circuit also satisfies eq. 1. However, due to the random number generator, the outputs are not related to the inputs and cannot be used for signaling. The dashed line on Bob's side now represents a trigger for a random number generator. The output on Bob's side does not depend on his input but equals the generated random number.

Note that now two bits are transmitted internally from Bob's side to Alice's side: one bit corresponds to the input of Bob and the second one to the result of the random number generator.

This PR-box is still asymmetric in the sense that Bob can perform his measurement and obtains an immediate output while Alice has to wait for Bob's input. In order to become a PR-box in the usual sense (except, as mentioned before, for the impossibility of superluminal internal signaling), Alice and Bob should be able to perform their measurements independently and obtain immediate outputs.

### 2.3 A PR-box with immediate outputs

Figure 3 shows a device which satisfies eq. (1), cannot be used for (external) signaling and allows both sides to perform their measurements with immediate results, independent of what happens at the other side. This is achieved by “symmetrizing” the box of the last section: whoever makes the first measurement triggers the random number generator and obtains the result of the random number generator as an output.

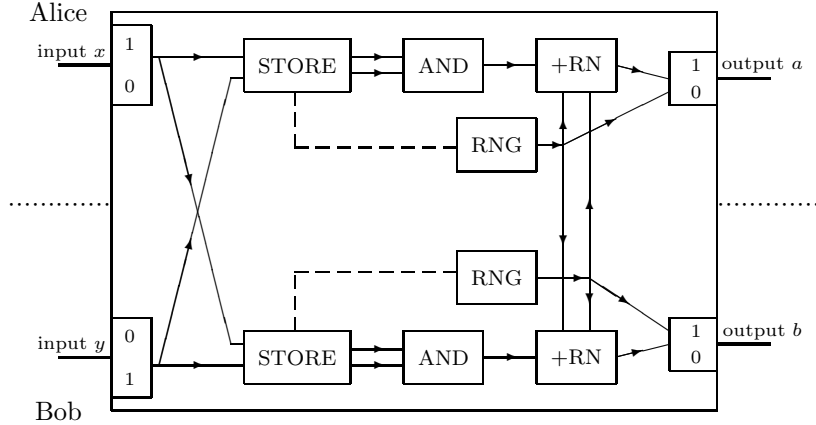


Figure 3: This PR-box yields immediate results when Alice and Bob perform their measurements. As before, dashed lines just trigger the random number generator RNG; they do not carry bits related to input or output. For details see main text.

Let us consider the device in more detail. The device is symmetric, but in order to be specific we assume that Alice makes the first measurement by inserting the input  $x$  into the box. This input  $x$  is sent to her STORE-gate which simply stores this input until also the second input  $y$  from Bob has entered. Furthermore,  $x$  is transmitted to Bob and stored in his STORE-gate. The STORE-gates “know” who has made the first measurement. For Alice, the STORE-gate blocks the further transmission of the input but instead triggers a

random number generator RNG which produces a signal and sends this signal to Alice’s output. She gets her immediate result. Furthermore, the random number generator sends its result to the “+RN”-device on Bob’s side which is stored there until Bob makes his measurement.

On Bob’s side the STORE-gate receives Alice’s input and now blocks the random number generator. Instead it waits for Bob’s input  $y$  (the second measurement). The result is transferred to the AND-gate which gives an output 1 if both  $x$  and  $y$  had been 1, otherwise it yields the output 0. The AND-gate transfers the result to Bob’s “+RN”-device where the random number which originally came from Alice is added. The result is transferred to Bob’s output who gets his result immediately.

Note that dashed lines in fig. 3 do not transmit any bits related to the inputs or outputs but only serve to trigger the random number generators. Furthermore, Alice and Bob should not perform their measurements within their light-cone complements as in this case both STORE-gates will “assume” that the measurement from the nearer side has been performed first and trigger the random number generators. In this case the outputs are random and do not satisfy the PR-box condition.

### 3 Reducing the internal exchange bits

PR-boxes are not meant to be built. However, it is instructive to see to which extent such a device can be realized as a classical physical system. The explicit realization – bound, however, to the constraints of special relativity – proves that such a box is not logically inconsistent.

The more interesting question is related to the number of bits which have to be exchanged internally between the two sides. As was to be expected, only one classical bit had to be exchanged in order to satisfy eq. 1, as this equation can be realized by a simple AND-gate. However, in order to make the device non-signalling (externally), it seems that at least one more classical bit has to be exchanged internally, which in the discussed case was the result of the random number generator.

This raises the natural question, whether the number of exchanged bits can be reduced if we use quantum correlations. Of course, it is known from superdense coding [3] that two classical bits can be exchanged by transmitting only one photon (which has an entangled partner on the other side). However, we also may replace the random number generator by an EPR state thus reducing the number of transferred classical bits to one.

The set-up would be as follows (Fig. 4): Both Alice and Bob share an entangled EPR state (e.g. realized by photons). Whoever makes the first measurement (say, Alice) by inserting her input bit  $x$ , triggers several events: first, a classical signal is sent to Bob transmitting the input of Alice. This signal is stored in the AND-gate of Bob until Bob makes his measurement. Furthermore, the input of  $x$  triggers on Alice’s side an EPR measurement on one of the entangled photons with respect to a predefined basis. The result (+1 or -1 translated into 1 and

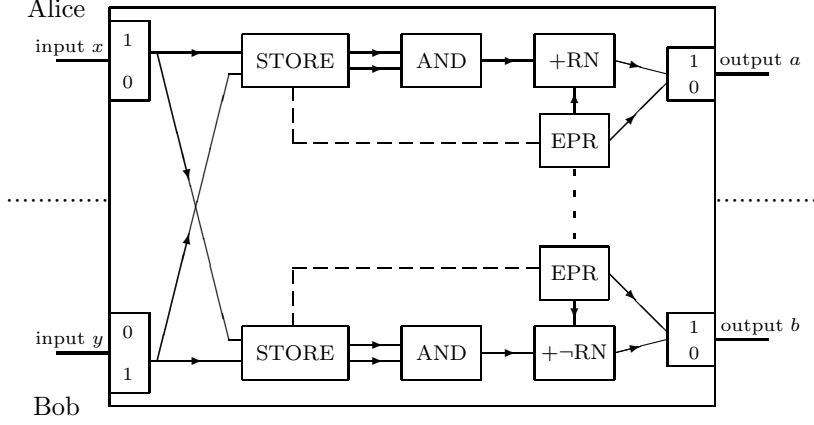


Figure 4: The random number generator of Fig. 3 has been replaced by an EPR-Bohm state of entangled particles shared by Alice and Bob. Internal measurements on this state can create correlated random numbers on Alice's and Bob's side without the exchange of information (because of the anti-correlation in EPR-states, Bob adds the negation of his EPR-result to the output of the AND-gate).

0, respectively) will be the result of Alice's measurement ( $a$ ). When the second measurement is performed (by Bob, inserting his input  $y$ ) internally two things happen: first, the input  $y$  is sent to Bob's AND-gate and processed with Alice's input  $x$  which is already there. Furthermore, on Bob's side a measurement is made on his EPR photon with respect to the same basis as Alice's measurement. If Alice and Bob have initially agreed to interpret the result of the EPR measurement (+1 and  $-1$ ) as 1 and 0, Bob has to reverse this result (because of the anti-correlations in the EPR states) before adding it to the output of the AND-gate in order to fulfill the PR-requirement. Hence, the EPR correlation reduces the two classical bits which have to be exchanged to one classical bit. In general, EPR correlations can be used to exchange random numbers without the exchange of classical bits.

Once more it should be emphasized that, although both sides can perform their measurements independently and obtain an immediate answer, the device does not work when both sides perform their measurements simultaneously within the causal complements of the light cones of each others measurements. In this case both sides trigger their random number generators and the answers will not satisfy the PR-box condition (eq. 1). It is known that a classical device with this property cannot exist.

## 4 Summary and Conclusion

I investigated several types of “real” PR-boxes of increasing complexity. Apart from measurements performed within causal complements, the constructed classical PR-box satisfies (1) the PR condition, (2) the no-signaling property, and (3) the possibility for independent measurements with immediate answers.

Classically it seems that in order to guarantee point (1) and (2), two classical bits have to be exchanged, of which the first one is an exchange of one of the inputs and the second one an exchange of a random number. The second classical bit can be replaced by a “non-signaling” EPR-measurement.

It has been shown that although there is internal signaling between the sides of Alice and Bob, the device cannot be used for external signaling. A violation of Bell-type inequalities together with the non-signaling property has been coined contextuality (see, e.g., [6]). So, what externally looks like contextuality may internally be due to signaling.

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